

# A five-dimensional effective model for excited light mesons

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## Abstract

In the usual holographic approach to QCD, the meson spectrum is generated due to a non-trivial 5-dimensional background. We propose an alternative 5-dimensional scenario in which the spectrum emerges due to coupling to a scalar field whose condensation is supposed to be dual to the formation of gluon condensate and mimics the scale anomaly in QCD. The spectrum of model has finite number of discrete states plus continuum and reveals a Regge-like behavior in the strong coupling regime.

## 1 Introduction

In the given report, we will discuss an effective five-dimensional model for excited light mesons (basing on our paper [1]) which represents an alternative to the standard bottom-up holographic approach to QCD [2]. The main difference consists in the mechanism for mass generation: Instead of mimicking the spectrum as Kaluza-Klein excitations of 5D fields living in a non-trivial 5D background we consider 5D fields in flat space coupled to a scalar field which acquires non-zero vacuum expectation value (v.e.v.) dependent on the fifth coordinate that has a physical meaning of inverse energy scale. The scalar field is supposed to be dual to the gluonic field strength tensor square. Such a construction mimics the scale anomaly in QCD and generates a discrete hadron spectrum with finite number of states plus continuum.

We will regard our model as an effective field theory that is valid below some energy scale, say below 2.5 GeV where the light mesons have been detected [3]. Staying within this viewpoint, we do not need the AdS metric in the UV limit as we do not perform matching to the UV asymptotics of QCD correlators and the simplest possibility will be considered — the flat metric. In addition, as is the case of many effective theories, our model is supposed to be applicable only on the tree level even in the strong coupling regime.

## 2 Vacuum sector

Consider a scalar field  $\varphi(x_\mu, z)$ ,  $\mu = 0, 1, 2, 3$ , living in five dimensions, where the 5th space-like coordinate  $z$  is the inverse energy scale,  $0 \leq z < \infty$ . Let us assume that this field is dual to the gluonic field strength tensor square  $G_{\mu\nu}^2$ . The latter is known to acquire a non-zero v.e.v.  $\langle G_{\mu\nu}^2 \rangle$  which causes the anomaly in the trace of energy-momentum tensor [4] in QCD,

$$4\varepsilon_{\text{vac}} = \langle \Theta_\mu^\mu \rangle_{\text{n.p.}} = \frac{\beta(\alpha_s)}{4\alpha_s} \langle G_{\mu\nu}^2 \rangle_{\text{n.p.}} + \mathcal{O}(\alpha) + \dots \quad (1)$$

where  $\varepsilon_{\text{vac}}$  denotes the vacuum energy, n.p. is "nonperturbative part" and the term  $\mathcal{O}(\alpha)$  is the contribution of quark polarization effects. We suppose that hadrons acquire observable masses due to interaction with the field  $\varphi$ . Taking the metric  $\eta_{AB} = (1, -1, -1, -1, -1)$ , the action describing the pure vacuum sector is postulated to be ( $A = 0, 1, 2, 3, 4$ )

$$S_{\text{CSB}} = \int d^4x dz \left( \frac{1}{2} \partial_A \varphi \partial^A \varphi + \frac{1}{2} m^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 \right). \quad (2)$$

By assumption, the self-interaction is weak,  $\lambda m \ll 1$ , hence, the semiclassical analysis may be applied.

The classical equation of motion is

$$\partial_\mu^2 \varphi - \partial_z^2 \varphi - \varphi(1 - \varphi^2) = 0. \quad (3)$$

We also assume that the v.e.v.  $\varphi_{\text{vac}}$  does not depend on the usual space-time coordinates,  $\varphi(x_\mu, z) = \varphi(z)$ . The equation (3) has then a kink solution

$$\varphi_{\text{vac}} = \pm \tanh(z/\sqrt{2}). \quad (4)$$

We choose the plus sign. The solution (4) breaks the translational invariance along the  $z$ -direction making thereby different energy scales non-equivalent. The given effect is important at large enough  $z$ , *i.e.* at low energies, at high energies the effect becomes negligible. This construction mimics the scale anomaly in QCD, with the dependence on the fifth coordinate in the solution  $\varphi_{\text{vac}}$  being related to the existence of anomalous dimension by the operator  $G_{\mu\nu}^2$ .

Consider the particle-like excitations of the vacuum state. Substituting  $\varphi = \varphi + \varepsilon$  into Eq. (3), retaining only linear in  $\varepsilon$  contributions and assuming  $\varepsilon(x_\mu, z) = e^{ipx}\varepsilon(z)$ , where  $p^2 = M^2$  is the usual 4D momentum defining the physical mass  $M$ , we obtain the following equation for the discrete mass spectrum,

$$\left(-\partial_z^2 + 3 \tanh^2(z/\sqrt{2}) - 1\right) \varepsilon_n = M_n^2 \varepsilon_n. \quad (5)$$

This one-dimensional Schrödinger equation can be easily solved analytically, it is known to have two normalizable discrete states (we omit the normalization factors),

$$\varepsilon_0 = \frac{1}{\cosh^2(z/\sqrt{2})}, \quad M_0^2 = 0; \quad (6)$$

$$\varepsilon_1 = \frac{\tanh(z/\sqrt{2})}{\cosh(z/\sqrt{2})}, \quad M_1^2 = \frac{3}{2}. \quad (7)$$

The continuum begins at  $p^2 = 2$ . The zero-frequency mode  $\varepsilon_0$  corresponds to the Goldstone boson of spontaneously broken translational invariance along the  $z$ -direction. This "dilaton" mode is located at high energies,  $z \rightarrow 0$ , where the scale invariance of the vacuum is restored asymptotically. An interesting feature of the model is the existence of a massive mode that is located near  $z_0 = \sqrt{2} \operatorname{arctanh}(1/\sqrt{2}) \approx 1.25$  and could be associated with a glueball according to the physical meaning of the field  $\varphi$ .

### 3 Adding bosons

Let us embed a massless hadron  $H$  in the vacuum. The interaction with the field  $\varphi$  should generate a certain mass for the hadron. We consider the simplest situation: The relevant action is quadratic in  $H$  and there is the factorization  $H(x_\mu, z) = H(x_\mu)H(z)$  that permits to obtain the particle-like excitations. Since the field  $H$  must be normalizable, we can then easily integrate over  $z$  and arrive at a 4D effective action. We will not introduce the couplings of bosons to the gradients of the scalar field  $\varphi$  since such vertices, although could play a role at high energies, are not important for the mass generation at low energies.

For simplicity, we analyse a scalar field  $\Phi$  coupled to the vacuum field  $\varphi$ , the extension of the analysis below to the higher spin fields is straightforward. The action is

$$S_{\text{bos}} = \int d^4x dz \left( \frac{1}{2} \partial_A \Phi \partial^A \Phi - \frac{G}{2} \varphi^2 \Phi^2 \right). \quad (8)$$

Consider the particle-like excitations  $\Phi(x_\mu, z) = e^{ipx}f(z)$ ,  $p^2 = M^2$  over the background (4). The classical equation of motion represents a one-dimensional Schrödinger equation

$$\left(-\partial_z^2 + \frac{G}{\lambda} \tanh^2(z/\sqrt{2})\right) f_n = M_n^2 f_n, \quad (9)$$

which determines the discrete spectrum

$$M_n^2 = \frac{1}{2} \left[ \sqrt{1 + \frac{8G}{\lambda}} \left( n + \frac{1}{2} \right) - \left( n + \frac{1}{2} \right)^2 - \frac{1}{4} \right], \quad (10)$$

$$f_n = \cosh^{n-s}(z/\sqrt{2}) F \left[ -n, 2s + 1 - n, s + 1 - n, \frac{1 - \tanh(z/\sqrt{2})}{2} \right] \quad (11)$$

where  $F$  is the hypergeometric function and

$$s = \frac{1}{2} \left( \sqrt{1 + \frac{8G}{\lambda}} - 1 \right), \quad n = 0, 1, 2, \dots, \quad n < s. \quad (12)$$

The continuum spectrum sets in at  $n = s$ . The states with  $n > 0$  carry the same quantum numbers as the ground state, in the language of potential models they are referred to as radial excitations. These excitations emerge if  $G/\lambda > 1$ , the number of the radial excitations is controlled by the value of coupling  $G/\lambda$  to the field  $\varphi$ . An interesting feature of obtained spectrum is that it is Regge-like,  $M_n^2 \sim n$ , in the strong coupling regime  $G/\lambda \gg 1$ . A numerical fit for the vector case (the experimental situation with the scalar mesons is ambiguous) is presented in Ref. [1]

## 4 Adding fermions

Consider massless fermions coupled to the scalar field  $\varphi$ . Following the standard procedure for introduction of fermions in 5D space, the simplest action is

$$S_{\text{ferm}} = \int d^4x dz (i\bar{\Psi}\Gamma^A\partial_A\Psi - h\varphi\bar{\Psi}\Psi), \quad (13)$$

where  $\psi$  is a four component spinor and  $\Gamma^\mu = \gamma^\mu$ ,  $\Gamma^4 = -i\gamma^5$ , here  $\gamma^\mu$ ,  $\gamma^5$  represent the usual 4D Dirac matrices. Let us find the particle-like excitations over the background (4). With the factorization  $\Psi_{L,R}(x_\mu, z) = e^{ipx}U_{L,R}(z)$  for the left and right components,  $\gamma_5\Psi_{L,R} = \pm\Psi_{L,R}$ , the relation for masses follows from the classical equation of motion,

$$\left( \pm\partial_z + \frac{h}{\sqrt{\lambda}} \tanh(z/\sqrt{2}) \right) U_{L,R} = MU_{L,R}. \quad (14)$$

At  $z \geq 0$ , the equation (14) possesses a normalizable zero-mode solution describing a massless left-handed fermion,

$$M = 0, \quad U_L = \cosh^{-\frac{\sqrt{2}h}{\sqrt{\lambda}}}(z/\sqrt{2}), \quad U_R = 0. \quad (15)$$

This mode is localized near  $z = 0$ . If we considered the region  $z < 0$  the situation would be opposite: The normalizable zero-mode solution describes a massless right-handed fermion. This feature is in agreement with the interpretation of negative-energy solutions of the Dirac equation as antiparticles. On the other hand, there is an asymptotic solution

$$z \rightarrow \infty : \quad M = \frac{h}{\sqrt{\lambda}}, \quad U_{L,R} = C_{L,R}. \quad (16)$$

Here  $C_{L,R}$  are some constants.

The generation of mass for the fermion in question can be obtained directly by means of integration over  $z$ . With the help of parametrization

$$\Psi(x_\mu, z) = s(z)\psi(x_\mu), \quad \int_0^\infty s^2(z)dz = 1, \quad (17)$$

we can integrate over  $z$  (the zero-mode will not contribute) and arrive at an effective 4D Lagrangian

$$\mathcal{L}_{\text{ferm}}^{(4)} = \bar{\psi}(i\gamma^\mu\partial_\mu - \mathcal{M})\psi, \quad (18)$$

with

$$\mathcal{M} = \frac{h}{\sqrt{\lambda}} \int_0^\infty s^2(z) \tanh(z/\sqrt{2}) dz. \quad (19)$$

We note that  $\mathcal{M} < h/\sqrt{\lambda}$  and the principal contribution to the effective mass  $\mathcal{M}$  comes from integration over large enough  $z$ , *i.e.* over the low-energy region.

Thus, the fermion described by Eq. (14) is massless and left-handed, this fermion is localized only at high energies, at low energies its wave function is suppressed exponentially; this phenomenon could mimic the confinement. In the limit of zero energy,  $z \rightarrow \infty$ , the massless fermion disappears and instead a massive fermion emerges that resembles a kink propagating like a particle. It is natural to associate this solution with a light quark in QCD which looks practically massless and left-handed if we probe it at high energies and acquires a constituent mass at low energies. After integration over the energy scales the zero-mode is lost and the model describes a massive fermion.

## 5 Discussions and outlook

The presented phenomenological model can serve as a basis for constructing various effective models for QCD. We have considered an application of the model to the calculation of boson masses and a coupling of fundamental quarks to a field mimicking the physical vacuum. The next natural applications of the proposed approach is the description of the baryon spectrum. A somewhat related question to be answered is the computing dynamical information encoded in the correlation functions and finding restrictions from the QCD sum rules in the narrow-width approximation (see, *e.g.*, [5] for a review).

The model has a natural limitation — it should be regarded as an effective model valid below some scale. As long as the model has only one massive excitation that might be associated with the scalar glueball, this scale could be the mass of the second scalar glueball. Since this mass is expected at approximately the same scale as the onset of perturbative continuum in light meson spectrum, about 2.5 GeV, the model is able to describe the whole discrete meson spectrum. As is the case of all effective models of QCD, the considered model has no well established relation to QCD itself, but rather mimics some features of the physics of strong interactions at low and intermediate energies.

The five-dimensional model of low-energy strong interactions studied here shares some general features with the bottom-up AdS/QCD models (see, *e.g.*, [2] and references in [6]). First of all, the 5th space dimension is introduced that has the physical meaning of inverse energy scale. The equations determining the mass spectrum of hadrons are reduced to the one-dimensional equations of the Schrödinger type. The holographic models, however, suffer from a large arbitrariness in the choice of background metric and boundary conditions on the holographic fields. As a consequence, any *ad hoc* spectrum can be obtained, in particular, the spectrum of the kind (10) [7]. The presented approach is much less arbitrary since the fifth coordinate is singled out dynamically, as a result one has a dynamical violation of the scale invariance while within the usual holographic models this is implemented "by hands". The considered approach represents thus a scheme for building the low-energy models of QCD in which the dependence on energy scale is included via the extension of usual space-time to the fifth "energetic" dimension, the latter can be always integrated out but we loose then the dynamical content of the theory. It turns out that the introduction of "energetic" space permits to calculate the spectrum of 4D theory and perhaps another static properties of hadrons in an essentially semiclassical way, such a possibility was the starting hypothesis of the AdS/QCD conjecture.

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## References

- [1] S. S. Afonin, *Int. J. Mod. Phys. A*, to be published; arXiv:0908.0457 [hep-ph].
- [2] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, *Phys. Rev. Lett.* **95**, 261602 (2005); G. F. de Teramond and S. J. Brodsky, *Phys. Rev. Lett.* **94**, 201601 (2005); L. Da Rold and A. Pomarol, *Nucl. Phys.* **B721**, 79 (2005); A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, *Phys. Rev.* **D74**, 015005 (2006).
- [3] C. Amsler *et al.*, *Phys. Lett.* **B667**, 1 (2008).
- [4] J.S. Collins, L. Duncan ,and S.D. Joglekar, *Phys. Rev.* **D16**, 438 (1977).
- [5] S. S. Afonin, A. A. Andrianov, V. A. Andrianov and D. Espriu, *JHEP* **0404**, 039 (2004).
- [6] S. S. Afonin, arXiv:1001.3105 [hep-ph].
- [7] S. S. Afonin, *Phys. Lett.* **B675**, 54 (2009).